On Refraction Tables.

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The operation of computing the difference between the true and the apparent zenith distances of stars, is usually performed by means of Bessel's refraction tables, which are calculated from his formula, namely :

$$R = d\ tan\zeta\ (BT)^{A}\gamma^{\lambda}\ *$$

This operation is usually logarithmic, and the refraction tables are therefore tables of logarithms. The computation may, however, be very much simplified, by the use of natural numbers in place of their logarithms, as will appear from the following :

In the above formula substitute R for $d\ tan\zeta$, and ρ for $(BT)^{A}\gamma^{\lambda}$. The formula then becomes

$$R = R_1\,\rho \ldots\ldots\ldots\ldots\ldots(1)$$

in which R_1 may be called the mean refraction, and ρ is a factor depending almost entirely on the temperature and pressure of the atmosphere. For general purposes two tables may be made, one of mean refractions (R_1), being the natural numbers corresponding to the table "$d\ tan\zeta$" of Bessel, and the other of corrections to the mean refraction. But since these corrections would be sometimes additive and sometimes subtractive, the latter table would be reduced to a smaller compass, if each observatory or place where the tables are adopted were to adapt them to the true mean refraction of that place. This may be done as follows:

Let ρ_0 be that value of ρ, which substituted in equation (1), would give true mean refraction for any particular place : and let R_0 be that mean refraction. Then

$$R_0 = R_1\ \rho_0 \ldots\ldots\ldots\ldots\ldots(2)$$

R_1 is given by its log. in Bessel's first table "*log.* $d\ tan\zeta$," and a round value for $log\ \rho_0$, near the above mentioned mean, may be chosen. From this formula a new table of mean refractions in natural numbers can easily be made.

Let r be the correction to be applied to this mean refraction. Then

$$r = R - R_0 = R_1\ (\rho - \rho_0) \ldots\ldots\ldots\ldots\ldots(3)$$

from which the table of corrections may be calculated.

*See Peter's "Astronomische Tafeln und Formeln."

In making the table, comparatively few values need be computed, for, having these few values, the others may be supplied by interpolation. The following is a small portion of a table of these corrections, in what appears to be its most convenient form. It was computed from Bessel's tables, as given by Peter's.

log. ρ	—0.01000	1100	1200	1300	1400	1500	&c.
ζ	Mean *R.*						
15°	15.12	0.03	0.07	0.10	0.14	0.17	
	106						
16	16.18	0.04	0.08	0.11	0.15	0.19	
	107						
17	17.25	0.04	0.08	0.12	0.16	0.20	
	108						
18	18.33	0.04	0.09	0.13	0.17	0.21	
	110						
19	19.43	0.04	0.09	0.13	0.18	0.22	
	111						
20	20.54	0.05	0.10	0.14	0.19	0.24	
	112						
21	21.66	0.05	0.10	0.15	0.20	0.25	
	114						
22°	22.80	0.05	0.11	0.16	0.21	0.26	
log. ρ	—0.01000	.00900	0801	0702	0604	0505	&c.

Apparent zenith distance is here used as argument at the side. It may be replaced, however, by the corresponding declination, or the circle reading. *Log.* ρ is used as argument at the top, and —0.01000 is the assumed value of *log.* ρ_0. When *log.* ρ = *log.* ρ_0 the correction is zero, therefore we have written the table of mean refractions in that column, thus combining both tables in one. This may be done when the arguments at the side are not chosen so far apart as to make the second differences of the mean refraction too great. In computing the tables from the above formulæ, the *arithmetical complement of log.* ρ must be used, when that *log.* as found from Bessel's tables, is negative. When *log.* ρ occurs in the line at the top of the table, the correction is negative, when in the line at the bottom, it is positive. To find the values of *log.* ρ for the lower line, corresponding to those of the upper line, call the natural number corresponding to any particular value in the upper line ρ_1, and its corresponding value for the lower line ρ_2. Then from formula (3), since r is minus in one case and plus in the other,

$$-\rho_1 + \rho_0 = \rho_2 - \rho_0$$

$$\text{or } \rho_2 = 2\rho_0 - \rho_1 \dots\dots\dots\dots (4)$$

Before entering the table of corrections, *log. B*, *log. T*, and *log.* γ are taken as usual from Bessel's tables, are then added, and their sum is the argument *log.* ρ, when the zenith distance is less than 45°. When the zenith distance is 45° or more, a correction on account of the exponent λ is to be applied to the log., and when 77° or more, another correction must be applied on account of the exponent A. These corrections may be found as follows :

Let $log.\ \rho_1 = log.\ (BT) + log.\ \gamma$, then the entire correction

$$r_1 = log.\ \rho - log.\ \rho_1 = log.\ \gamma\ (\lambda - 1) + log.\ (BT)\ (A - 1) \ldots . (5)$$

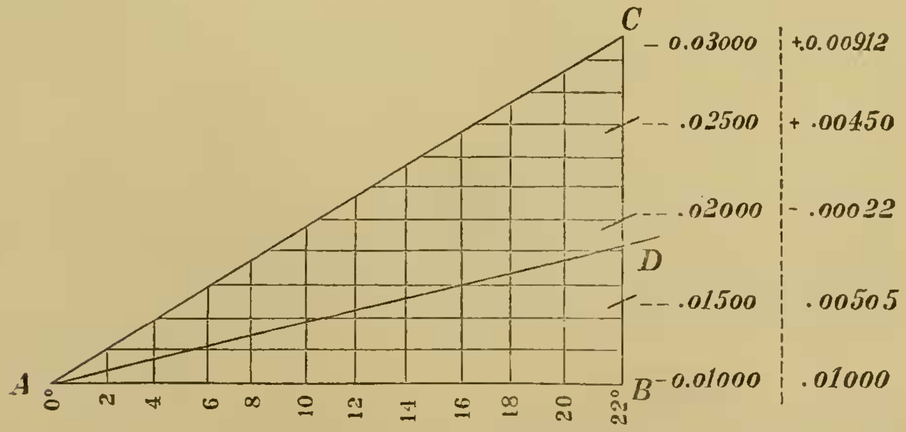
in which the two terms of the last member are the first and second corrections respectively. The following will illustrate a method of tabulating these corrections :

ζ	0.01000	2000	3000	4000	ζ
60°	5	9	14	18	80° 40′
61	5	10	15	20	81 0
62	5	11	16	22	81 30
63	6	12	17	23	81 50
64	6	13	19	25	82 10
65°	7	14	20	27	82° 30′

The argument at the top is either $log.\ (BT)$ or $log.\ \gamma$, according to which correction is sought for. The side arguments are zenith distances, which may be replaced by declinations, &c.

The first column is used when taking out the first correction, and the last column, when taking out the second correction. To find the values of the argument for the last column, enter Bessel's table "A" with the value of λ for any zenith distance of the first column, and against it will be found the corresponding zenith distance for the last column. The units place of the corrections, corresponds to the fifth place of $log.\ \rho$, and when the logarithmic argument is negative, the correction is negative, and *vice versa.*

In place of the above table of corrections to mean refraction, a graphical table may be constructed, which has some advantages. Let a right-angle triangle $A\ B\ C$ be drawn, containing lines parallel to the base, at equal distances apart.



These equal distances may represent seconds, or tenths of a second. The distance $B\ C$ is made equal to the maximum correction rm, or the correction for the maximum zenith distance, and maximum value of $log.\ \rho$, which occur in the table. Values of r are now computed for the different values of the argument $log. \, _{\prime}\rho$, but for the maximum zenith distance, and the results laid off on the line $B\ C$, from the point B; and against the points thus found the arguments $log \, _{\prime}\rho$ are written. Along the line AB are written the zenith distances. Equal increments of zenith distance are represented by unequal increments of the line AB, in order that the line AC, or line of maximum corrections, may be a straight line. Call the co-ordinates of any point of AC, d and r, and the distance AB, dm. Then

$$d : dm = r : rm$$

$$\text{or } d = \frac{dm}{rm} r = \frac{dm}{rm} R_1 \left(_{\prime}\rho m - _{\prime}\rho_0 \right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

from which the distances of the arguments from the point A may be determined. The line AD represents the edge of a ruler, or a thread, stretched from a needle at the point A. If this line be turned about A, until it passes through that point BC, corresponding to the value of $log \, _{\prime}\rho$ for which the correction is sought, the correction will be the length of the perpendicular to AB, measured to its intersection with AD, at the point corresponding to the given zenith distance.

This method has the advantage that the interpolations are made separately, and will undoubtedly give good results, if the diagram is drawn carefully, and large enough.